Second Generation Panel Unit Root Tests

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Abstract

This article proposes an overview of the recent developments relating to panel unit root tests. After a brief review of the first generation panel unit root tests, this paper focuses on the tests belonging to the second generation. The latter category of tests is characterized by the rejection of the cross-sectional independence hypothesis. Within this second generation of tests, two main approaches are distinguished. The first one relies on the factor structure approach and includes the contributions of Bai and Ng (2001), Phillips and Sul (2003a), Moon and Perron (2004a), Choi (2002) and Pesaran (2003) among others. The second approach consists in imposing few or none restrictions on the residuals covariance matrix and has been adopted notably by Chang (2002, 2004), who proposed the use of nonlinear instrumental variables methods or the use of bootstrap approaches to solve the nuisanceparameter problem due to cross-sectional dependency.

- *Keywords*: Nonstationary panel data, unit root, heterogeneity, cross-sectional dependencies.

- *J.E.L Classification* : C23, C33

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1 Introduction

Since the seminal works of Levin and Lin (1992, 1993) and Quah (1994), the study of unit roots has played an increasingly important role in empirical analysis of panel data. Indeed, the investigation of integrated series in panel data has known a great development and panel unit root tests have been applied to various fields of economics: analysis of the purchasing power parity hypothesis, growth and convergence issues, saving and investment dynamics, international R&D spillovers, etc[1].

Adding the cross-sectional dimension to the usual time dimension is very important in the context of nonstationary series. Indeed, it is well known that unit root tests generally have low power in small sample sizes to distinguish nonstationary series from stationary series that are persistent. In order to increase the power of unit root tests, a solution is to increase the number of observations by including information relating to various individuals or countries. Thus, the use of panel data allows to solve the low power issue of unit root tests in small samples by increasing the number of observations. As noted by Baltagi and Kao (2000), the econometrics of nonstationary panel data aims at combining *"the best of both worlds: the method of dealing with nonstationary data from the time series and the increased data and power from the cross-section"*.

The first main difference between unit root tests in time series data and panel data concerns the issue of heterogeneity. In the time series case, heterogeneity is not a problem since the unit root hypothesis is tested in a given model for a given individual. Things are different in a panel data context: can we consider the same model for testing the unit root hypothesis on various individuals? A positive answer means that the panel is homogeneous. But, if individuals are characterized by different dynamics, the panel is heterogeneous and the panel unit root tests must take into account this heterogeneity, even if tests based on pooled estimates of the autoregressive parameters could be consistent against a heterogeneous alternative (Moon and Perron, 2004b). This notion of heterogeneity constitutes a central point in the econometrics of panel data (Hsiao, 1986, Pesaran and Smith, 1995 for the dynamic models). Naturally, the issue of the specification of the alternative constituted the first departure in the literature. After the seminal paper proposed by Levin and Lin (1992, 1993) and Levin, Lin and Chu (2002) based on a pooled estimator of the autoregressive parameter, numerous tests based on a heterogeneous specification of the alternative have been proposed by Im, Pesaran and Shin (1997), Maddala and Wu (1999), Choi (2001) and Hadri (2000) for instance.

Besides this evolution towards heterogeneous specifications, a second evolution has been recently observed and concerns the existence of cross-sectional dependencies. According to whether unit root tests allow for potential correlations across residuals of panel units, two generations of tests can be distinguished, as listed in table 1.

The first generation of panel unit root tests is based on the cross-sectional independency hypothesis: Levin and Lin (1992, 1993), Levin *et al.* (2002), Harris and Tzavalis (1999), Im *et al.* (1997, 2002, 2003), Maddala and Wu (1999), Choi (1999, 2001), Hadri (2000). Within this

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[1] For a survey, see Banerjee (1999) and Baltagi and Kao (2000).

Table 1: Panel Unit Root Tests

First Generation	Cross-sectional independence
1. *Nonstationarity tests*	Levin and Lin (1992, 1993)
	Levin, Lin and Chu (2002)
	Harris and Tzavalis (1999)
	Im, Pesaran and Shin (1997, 2002, 2003)
	Maddala and Wu (1999)
	Choi (1999, 2001)
2- *Stationarity tests*	Hadri (2000)
Second Generation	Cross-sectional dependencies
1- *Factor structure*	Bai and Ng (2001, 2004)
	Moon and Perron (2004a)
	Phillips and Sul (2003a)
	Pesaran (2003)
	Choi (2002)
2- *Other approaches*	O'Connell (1998)
	Chang (2002, 2004)

context, correlations across units constitute nuisance parameters. The cross-sectional independency hypothesis is rather restrictive and somewhat unrealistic in the majority of macroeconomic applications of unit root tests, like the study of convergence (Phillips and Sul, 2003b) or the analysis of purchasing power parity (O'Connell, 1998) where co-movements of economies are often observed.

This is an important issue since the application of tests belonging to the first generation to series that are characterized by cross-sectional dependencies leads to size distortions and low power (Banerjee, Marcellino and Osbat, 2000, Strauss and Yigit, 2003). In response to the need for panel unit root tests that allows for cross-sectional correlations, various tests have been proposed belonging to what we call the class of the second generation tests. Rather than considering correlations across units as nuisance parameters, this new category of tests aims at exploiting these co-movements in order to define new test statistics. As argued by Quah (1994), the modelling of cross-sectional dependencies is a difficult task since no natural ordering exists in unit observations. This is why various tests have been proposed including the works of Bai and Ng (2001), Phillips and Sul (2003a), Moon and Perron (2004a), Choi (2002), Ploberger and Phillips (2002), Moon, Perron and Phillips (2003), Chang (2002) and Pesaran (2003). Two main approaches can be distinguished. The first one relies on the factor structure approach and includes the contributions of Bai and Ng (2001), Phillips and Sul (2003a), Moon and Perron (2004a), Choi (2002) and Pesaran (2003). The second approach consists in imposing few or none restrictions on the residuals covariance matrix. This approach has been adopted by Chang (2002) among others, who proposed the use of instrumental variables in order to solve the nuisance parameter problem due to cross-sectional dependency.

The paper is organized as follows. Section 2 is devoted to a brief overview of first generation panel unit root tests. In section 3, we present the second generation of tests based on the factor structure approach. In section 4, other approaches that belong to the class of second generation tests are presented. Section 5 concludes.

2 A first generation of tests

The first generation of panel unit root tests is based on the cross-sectional independence assumption and includes the contributions of Levin and Lin (1992, 1993), Im *et al.* (1997, 2003), Maddala and Wu (1999), Choi (2001) and Hadri (2000).

2.1 The Levin and Lin tests

The first tests presented in this paper are the Levin and Lin (LL thereafter) tests proposed in Levin and Lin (1992, 1993) and Levin, Lin and Chu (2002). Let us consider a variable observed on N countries and T periods and a model with individual effects and no time trend. As it is well known, LL consider a model in which the coefficient of the lagged dependent variable is restricted to be homogenous across all units of the panel:

$$\Delta y_{i,t} = \alpha_i + \rho\, y_{i,t-1} + \sum_{z=1}^{p_i} \beta_{i,z} \Delta y_{i,t-z} + \varepsilon_{i,t} \tag{1}$$

for $i = 1,...,N$ and $t = 1,...,T$. The errors $\varepsilon_{i,t}$ *i.i.d.* $\left(0, \sigma_{\varepsilon_i}^2\right)$ are assumed to be independent across the units of the sample. In this model, LL are interested in testing the null hypothesis $H_0 : \rho = 0$ against the alternative hypothesis $H_1 : \rho = \rho_i < 0$ for all $i = 1,...,N$, with auxiliary assumptions about the individual effects ($\alpha_i = 0$ for all $i = 1,...,N$ under H_0). This alternative hypothesis is restrictive since it implies that the autoregressive parameters are identical across the panel. As pointed out by Maddala and Wu (1999), it is too strong to be held in any interesting empirical cases. For instance, in testing the convergence hypothesis in growth models, the LL alternative restricts every country to converge at the same rate.

However it is important to note that the use of a pooled estimator $\widehat{\rho}$, even if the genuine DGP is heterogeneous, does not imply that the unit root test is not consistent. As it will be the case for the test of second generation of Moon and Perron (2004a) also based on a pooled estimator, the LL test is consistent against a heterogeneous alternative hypothesis[2]. For instance, let us consider a simple linear model $y_i = x_i \beta_i + \varepsilon_i$ in which the parameter β_i is equal to 0 for one half of the sample and 1 for the other half. Let us assume that we want to test the null $\beta_i = 0$ for all the units. It is possible to use in this context a pooled estimate of $\widehat{\beta}$ on the whole sample. The pooled OLS estimator will converge to 0.5 and the standard error will converge to zero. Then, the null will be rejected. Naturally, it is possible to get a more powerful test by splitting the sample in two parts and testing the null in both parts. So, it is important to separate the issue of estimating the value of the autoregressive parameter (in order to estimate the rate of convergence for instance) and the issue of testing its value.

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[2] See Moon and Perron (2004b) for a more general discussion on the asymptotic local power of pooled t-ratio tests against homogeneous or heterogeneous alternatives.

In a model without deterministic component, under the null, the standard t-statistic t_ρ based on the pooled estimator $\widehat{\rho}$ has a standard normal distribution when N and T tend to infinity and $\sqrt{N}/T \to 0$. However this statistic diverges to negative infinity in a model with individual effects. That is why, LL suggest using the following adjusted t-statistic:

$$t_\rho^* = \frac{t_\rho}{\sigma_T^*} - N T \widehat{S}_N \left(\frac{\widehat{\sigma}_{\widehat{\rho}}}{\widehat{\sigma}_{\widehat{\varepsilon}}^2} \right) \left(\frac{\mu_T^*}{\sigma_T^*} \right) \tag{2}$$

where the mean adjustment μ_T^* and standard deviation adjustment σ_T^* are simulated by authors (Levin, Lin and Chu, 2002, table 2) for various sample sizes T. The adjustment term is also function of the average of individual ratios of long-run to short-run variances, $\widehat{S}_N = (1/N) \sum_{i=1}^N \widehat{\sigma}_{y_i}/\widehat{\sigma}_{\varepsilon_i}$, where $\widehat{\sigma}_{y_i}$ denotes a kernel estimator of the long-run variance for the country i. LL suggest using a Bartlett kernel function and a homogeneous truncation lag parameter given by the simple formula $\overline{K} = 3.21 T^{1/3}$. They demonstrate that, provided the maximum ADF (Augmented Dickey-Fuller) lag order increased at some rate T^p where $0 < p \leq 1/4$ and the lag truncation parameter $\overline{K}$ increased at rate T^q where $0 < q < 1$, the adjusted t-statistic t_ρ^* converges to a standard normal distribution under the nonstationary null hypothesis:

$$t_\rho^* \xrightarrow[N,T \to \infty]{d} N(0,1) \quad \text{with } \sqrt{N}/T \to \infty \tag{3}$$

2.2 The Im, Pesaran and Shin tests

The second test based on the cross-sectional independence assumption presented in this study is the well-known Im, Pesaran and Shin test (1997, 2003), IPS thereafter. On the contrary to LL, this test allows for heterogeneity in the value of ρ_i under the alternative hypothesis. IPS consider the model (1) and substitute ρ_i for ρ. Their model with individual effects and no time trend is now:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \sum_{z=1}^{p_i} \beta_{i,z} \Delta y_{i,t-z} + \varepsilon_{i,t} \tag{4}$$

The null hypothesis[3] is defined as $H_0 : \rho_i = 0$ for all $i = 1, ..., N$ and the alternative hypothesis is $H_1 : \rho_i < 0$ for $i = 1, ..., N_1$ and $\rho_i = 0$ for $i = N_1 + 1, ..., N$, with $0 < N_1 \leq N$. The alternative hypothesis allows for some (but not all) of the individual series to have unit roots. Thus, instead of pooling the data, IPS use separate unit root tests for the N cross-section units. Their test is based on the (augmented) Dickey-Fuller statistics averaged across groups. Let $t_{iT}(p_i, \beta_i)$ with $\beta_i = (\beta_{i,1}, ..., \beta_{i,p_i})$ denote the t-statistic for testing unit root in the ith country, the IPS statistic is then defined as:

$$t\_bar_{NT} = \frac{1}{N} \sum_{i=1}^N t_{iT}(p_i, \beta_i) \tag{5}$$

As it was previously mentioned, under the crucial assumption of cross-sectional independence, this statistic is shown to sequentially converge to a normal distribution when T tends to infinity, followed by N. The intuition is as follows. When T tends to infinity, each individual

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[3] The null hypothesis in IPS also implies auxiliary assumptions about the individual effects as in LL and in particular $\alpha_i = 0$ for all $i = 1, ..., N$.

statistic $t_{iT}(p_i, \beta_i)$ converges to the Dickey-Fuller distribution. If we assume that the residuals $\varepsilon_{i,t}$ and $\varepsilon_{j,s}$ are independent for $i \neq j$, $\forall\, (t,s)$, the corresponding statistics $t_{iT}(p_i, \beta_i)$ and $t_{jT}(p_j, \beta_j)$ are also independent for all T. So, when T tends to infinity, the individual statistics $t_{iT}(p_i, \beta_i)$ are independently and identically distributed. The use of a simple Lindberg-Levy central limit theorem is then sufficient to show that the cross-sectional average mean t_bar_{NT} converges to a normal distribution when N tends to infinity. It clearly shows that the cross-sectional independence assumption is the central assumption to establish the normal limiting distribution of the IPS statistic. A similar result is conjectured when N and T tend to infinity while the ratio N/T tends to a finite non-negative constant.

In order to propose a standardization of the t-bar statistic, IPS have to compute the values of $E\left[t_{iT}(p_i, \beta_i)\right]$ and $Var\left[t_{iT}(p_i, \beta_i)\right]$. When T tends to infinity these moments tend to the corresponding moments of the Dickey-Fuller distribution, i.e. $E(\eta) = -1.532$ and $Var(\eta) = 0.706$ (Nabeya, 1999). However, in the case of serial correlation, when T is fixed, these moments will depend on the nuisance parameters β_i even under the null hypothesis $\rho_i = 0$. Therefore, the standardization will not be practical. Two solutions can be considered: the first one is based on the asymptotic moments $E(\eta)$ and $Var(\eta)$. The corresponding standardized t-bar statistic is denoted $Z_{t\,bar}$. The second solution of practical relevance is to carry out the standardization of the t-bar statistic using the means and variances of $t_{iT}(p_i, 0)$ evaluated by simulations under the null $\rho_i = 0$. IPS conjecture that the corresponding standardized t-bar statistic, denoted W_{tbar}, converges to a standard normal distribution under the null of nonstationarity along the diagonal $N/T \to k$, with $k > 0$:

$$W_{tbar} = \frac{\sqrt{N}\left[t\_bar_{NT} - N^{-1}\sum_{i=1}^{N} E\left[t_{iT}(p_i, 0)|\,\rho_i = 0\right]\right]}{\sqrt{N^{-1}\sum_{i=1}^{N} Var\left[t_{iT}(p_i, 0)|\,\rho_i = 0\right]}} \xrightarrow[T,N\to\infty]{d} N(0,1) \qquad (6)$$

Although the tests Z_{tbar} and W_{tbar} are asymptotically equivalent, simulations show that the W_{tbar} statistic — which explicitly takes into account the underlying ADF orders in computing the mean and the variance adjustment factors — performs much better in small samples.

2.3   The Fisher's type tests : Maddala and Wu (1999) and Choi (2001)

As it was mentioned in the previous section, the panel unit root tests based on a heterogeneous model consist in testing the significance of the results from N independent individual tests. In this context, IPS use an average statistic, but there is an alternative testing strategy based on combining the observed significant levels from the individual tests. This approach based on p-values has a long history in meta-analysis. In panel unit root tests, such a strategy based on Fisher (1932) type tests, was notably used by Choi (2001) and Maddala and Wu (1999).

Let us consider a heterogeneous model (equation 4). We test the same hypothesis as IPS, $H_0 : \rho_i = 0$ for all $i = 1, ..., N$ against the alternative hypothesis $H_1 : \rho_i < 0$ for $i = 1, ..., N_1$ and $\rho_i = 0$ for $i = N_1 + 1, ..., N$, with $0 < N_1 \leq N$. The idea of the Fisher type test is very simple. Let us consider pure time series unit root test statistics (ADF, Elliott-Rothenberg-Stock, Max-ADF, *etc.*). If these statistics are continuous, the corresponding p-values, denoted

p_i, are uniform $(0, 1)$ variables. Consequently, under the crucial assumption of cross-sectional independence, the statistic proposed by Maddala and Wu (1999) defined as:

$$P_{MW} = -2 \sum_{i=1}^{N} \log(p_i) \tag{7}$$

has a chi-square distribution with $2N$ degrees of freedom, when T tends to infinity and N is fixed. As noted by Banerjee (1999), the obvious simplicity of this test and its robustness to statistic choice, lag length and sample size make it extremely attractive. For large N samples, Choi (2001) proposes a similar standardized[4] statistic:

$$Z_{MW} = \frac{\sqrt{N} \left\{ N^{-1} P_{MW} - E\left[-2\log(p_i)\right] \right\}}{\sqrt{Var\left[-2\log(p_i)\right]}} = -\frac{\sum_{i=1}^{N} \log(p_i) + N}{\sqrt{N}} \tag{8}$$

This statistic corresponds to the standardized cross-sectional average of individual p-values. Under the cross-sectional independence assumption, the Lindberg-Levy theorem is sufficient to show that it converges to a standard normal distribution under the unit root hypothesis.

2.4 The Hadri test

Contrary to the previous first generation tests, the test proposed by Hadri (2000) is based on the null hypothesis of stationarity. It is an extension of the stationarity test developed by Kwiatkowski *et al.* (1992) in the time series context. Hadri proposes a residual-based Lagrange multiplier test for the null hypothesis that the individual series $y_{i,t}$ (for $i = 1, ..., N$) are stationary around a deterministic level or around a deterministic trend, against the alternative of a unit root in panel data. Hadri (2000) considers the two following models:

$$y_{i,t} = r_{i,t} + \varepsilon_{i,t} \tag{9}$$

and

$$y_{i,t} = r_{i,t} + \beta_i t + \varepsilon_{i,t} \tag{10}$$

where $r_{i,t}$ is a random walk: $r_{i,t} = r_{i,t-1} + u_{i,t}$, $u_{i,t}$ is $i.i.d.$ $\left(0, \sigma_u^2\right)$, $u_{i,t}$ and $\varepsilon_{i,t}$ being independent. The null hypothesis can thus be stated as: $\sigma_u^2 = 0$. Moreover, since the $\varepsilon_{i,t}$ are assumed $i.i.d.$, then, under the null hypothesis, $y_{i,t}$ is stationary around a deterministic level in model (9) and around a deterministic trend in model (10). Model (9) can also be written:

$$y_{i,t} = r_{i,0} + e_{i,t} \tag{11}$$

and model (10):

$$y_{i,t} = r_{i,0} + \beta_i t + e_{i,t} \tag{12}$$

with $e_{i,t} = \sum_{j=1}^{t} u_{i,j} + \varepsilon_{i,t}$, $r_{i,0}$ being initial values that play the role of heterogeneous intercepts.

It should be noted that if $\sigma_u^2 = 0$, then $e_{i,t} \equiv \varepsilon_{i,t}$ is stationary ($r_{i,t}$ is a constant). If $\sigma_u^2 \neq 0$, $e_{i,t}$ is nonstationary ($r_{i,t}$ is a random walk). More specifically, Hadri (2000) tests the null $\lambda = 0$

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[4] Since $E\left[-2\log(p_i)\right] = 2$ and $Var\left[-2\log(p_i)\right] = 4$.

against the alternative $\lambda > 0$ where $\lambda = \sigma_u^2/\sigma_\varepsilon^2$. Let $\hat{e}_{i,t}$ be the estimated residuals from (11) or (12), the LM statistic is given by:

$$LM = \frac{1}{\hat{\sigma}_\varepsilon^2} \frac{1}{NT^2} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} S_{i,t}^2 \right) \tag{13}$$

where $S_{i,t}$ denotes the partial sum of the residuals: $S_{i,t} = \sum_{j=1}^{t} \hat{e}_{i,j}$ and $\hat{\sigma}_\varepsilon^2$ is a consistent estimator of σ_ε^2. Under the null of level stationarity (model (9)), the test statistic:

$$Z_\mu = \frac{\sqrt{N} \left\{ LM - E\left[\int_0^1 V(r)^2 dr \right] \right\}}{\sqrt{V\left[\int_0^1 V(r)^2 dr \right]}} \tag{14}$$

follows a standard normal law, where $V(r)$ is a standard Brownian bridge, for $T \to \infty$ followed by $N \to \infty$. The cumulants of the characteristic function of $\int_0^1 V^2$ give, respectively, the mean and the variance of $\int_0^1 V(r)^2$ in (14): $1/6$ (first cumulant) for the mean and $1/45$ (second cumulant) for the variance (see Hadri, 2000, for details)[5].

3 A second generation of tests: the factor structure approach

The second generation unit root tests relax the cross-sectional independence assumption. The issue is to specify these cross-sectional dependencies. As mentioned above, this specification is not obvious since individual observations in a cross-section have no natural ordering, except if we consider a metric of economic distance. So, in response to the need for panel unit root tests allowing cross-sectional correlation, researchers have devised various methods. Two groups can be distinguished: in the first group the cross-sectional dependencies are specified as a common factor model. The second group will be studied in the next section.

3.1 The Bai and Ng tests

Bai and Ng (2001, 2004) have proposed the first test of the unit root null hypothesis taking into account the potential cross-sectional correlation. The problem consists on the specification of a special form of these dependences. Bai and Ng suggest a rather simple approach and consider a factor analytic model:

$$y_{i,t} = D_{i,t} + \lambda_i' F_t + e_{i,t} \tag{15}$$

where $D_{i,t}$ is a polynomial time function of order t, F_t is a $(r,1)$ vector of common factors, and λ_i is a vector of factor loadings. Thus, the individual series $y_{i,t}$ is decomposed into a heterogeneous deterministic component $D_{i,t}$, a common component $\lambda_i' F_t$ and an error term $e_{i,t}$ largely idiosyncratic. It is worth noting that it is the presence of the common factors F_t, according to which each individual has a specific elasticity λ_i, which is at the origin of the cross-sectional dependencies.

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[5] In presence of a deterministic trend, $V(r)$ should be replaced by $V_2(r)$ in (14) where $V_2(r) = W(r) + (2r - 3r^2)W(1) + 6r(r-1)\int_0^1 W(s)ds$ (see Kwiatkowski *et al.* (1992)). The mean and the variance of $\int_0^1 V_2^2$ are given by the first two cumulants: respectively, $1/15$ and $11/6300$.

In this case, $y_{i,t}$ is said to be nonstationary if at least one common factor of the vector F_t is nonstationary and/or the idiosyncratic term $e_{i,t}$ is nonstationary. Nothing guarantees that these two terms have the same dynamic properties: one could be stationary, the other nonstationary, some components of F_t could be $I(0)$, others $I(1)$, F_t and $e_{i,t}$ could be integrated of different orders, *etc*. However, it is well known that a series defined as the sum of two components which have different dynamic properties has itself dynamic properties which are very different from its entities. Thus, it may be difficult to check the stationarity of $y_{i,t}$ if this series contains a large stationary component. This is why, rather than directly testing the nonstationarity of $y_{i,t}$, Bai and Ng (2004) suggest to separately test the presence of a unit root in the common and individual components. This procedure is called PANIC (*Panel Analysis of Nonstationarity in the Idiosyncratic and Common components*) by the authors.

What is the advantage of this procedure in regard to the cross-sectional dependencies? It stays in the fact that the idiosyncratic component $e_{i,t}$ can be considered as being slightly correlated across individuals, while, at the same time, the complete series $y_{i,t}$ can present high cross-sectional correlations. One of the main critics addressed to the first generation of unit root tests, principally in the context of macroeconomic series, is thus dropped.

Suppose that the deterministic component $D_{i,t}$ can be represented by individual effects α_i without time trend. Then, it is possible to write the model as follows:

$$y_{i,t} = \alpha_i + \lambda_i' F_t + e_{i,t}, \quad t = 1, ..., T \tag{16}$$

$$F_{m,t} = \tau_m F_{m,t-1} + v_{m,t}, \quad m = 1, ..., r \tag{17}$$

$$e_{i,t} = \rho_i e_{i,t-1} + \varepsilon_{i,t}, \quad i = 1, ..., N \tag{18}$$

The mth common factor $F_{m,t}$ is stationary if $\tau_m < 1$. The idiosyncratic component $e_{i,t}$ is stationary for the ith individual if $\rho_i < 1$. The objective is to apprehend the stationarity of $F_{m,t}$ and $e_{i,t}$ given that these components are not observed and must be estimated. The validity of PANIC thus depends on the possibility to obtain estimators of $F_{m,t}$ and $e_{i,t}$ that preserve their integration degree, whatever $e_{i,t}$ is $I(0)$ or $I(1)$. In others words, the common variations must be extracted without appealing to stationarity assumptions and/or cointegration restrictions. Bai and Ng accomplish this by estimating factors on first-differenced data and cumulating these estimated factors. Let us assume that the number of common factors r in the first differences is known[6].

Consider the model in its first-differenced form:

$$\Delta y_{i,t} = \lambda_i' f_t + z_{i,t} \tag{19}$$

where $z_{i,t} = \Delta e_{i,t}$ and $f_t = \Delta F_t$ is such that $E(f_t) = 0$. Moreover, consider the following notations:

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[6] For an estimation of r, see Bai and Ng (2002).

$$
\underset{(N,r)}{\Lambda} = \begin{pmatrix} \underset{(1,r)}{\lambda_1'} \\ .. \\ \underset{(1,r)}{\lambda_N'} \end{pmatrix} \qquad \underset{(T-1,r)}{f} = \begin{pmatrix} \underset{(1,r)}{f_2'} \\ \underset{(1,r)}{f_3'} \\ .. \\ \underset{(1,r)}{f_T'} \end{pmatrix} \qquad \underset{(T-1,N)}{X} = \begin{pmatrix} \Delta y_{1,2} & .. & \Delta y_{N,2} \\ \Delta y_{1,3} & & \Delta y_{N,3} \\ ... & & \\ \Delta y_{1,T} & & \Delta y_{N,T} \end{pmatrix}
$$

The Bai and Ng testing procedure implies, in a first step, to estimate the common factors in $\Delta y_{i,t}$ by a simple principal component method. Then the estimate $\widehat{f}$ of the f matrix is $\sqrt{T-1}$ times a matrix whose columns are defined by the r eigenvectors associated with the r largest eigenvalues of the XX' matrix. The estimated loading (N,r) matrix $\widehat{\Lambda}$ is defined by $\widehat{\Lambda} = X'\widehat{f}/(T-1)$. One denotes $\widehat{z}_{i,t} = \Delta y_{i,t} - \widehat{\lambda}_i'\widehat{f}_t$. Then, the 'differencing and re-cumulating' estimation procedure is based on the cumulated variables defined as:

$$
\hat{F}_{m,t} = \sum_{s=2}^{t} \hat{f}_{m,s} \qquad \hat{e}_{i,t} = \sum_{s=2}^{t} \widehat{z}_{i,s} \tag{20}
$$

for $t = 2,...,T$, $m = 1,...,r$ and $i = 1,...,N$. Bai and Ng test the unit root hypothesis in the idiosyncratic component $e_{i,t}$ and in the common factors F_t with the estimated variables $\hat{F}_{m,t}$ and $\hat{e}_{i,t}$.

To test the nonstationarity of the idiosyncratic component, Bai and Ng propose to pool individual ADF t-statistics computed with the de-factored estimated components $\hat{e}_{i,t}$ in a model with no deterministic term:

$$
\Delta\hat{e}_{i,t} = \delta_{i,0}\hat{e}_{i,t-1} + \delta_{i,1}\Delta\hat{e}_{i,t-1} + .. + \delta_{i,p}\Delta\hat{e}_{i,t-p} + \mu_{i,t} \tag{21}
$$

Let $ADF_{\hat{e}}^c(i)$ be the ADF t-statistic for the idiosyncratic component of the ith country. The asymptotic distribution of $ADF_{\hat{e}}^c(i)$ coincides with the Dickey-Fuller distribution for the case of no constant. Therefore, a unit root test can be done for each idiosyncratic component of the panel. The great difference with unit root tests based on the pure time series is that the common factors, as global international trends or international business cycles for instance, have been withdrawn from data.

However, these tests implemented on time series in the last step have low power in small sample sizes T. For this reason, Bai and Ng propose the use of a mean statistic *à la* Im, Pesaran and Shin (2003), or a statistic *à la* Maddala et Wu (1999) such that:

$$
Z^c = \frac{1}{\sqrt{4N}} \left[-2\sum_{i=1}^{N} \log p^c(i) - 2N \right] \tag{22}
$$

where $p^c(i)$ is the p-value associated with the statistic $ADF_e^c(i)$. At this stage, Bai and Ng have to suppose cross-sectional independency between unobservable individual components $e_{i,t}$ (theorem 3, page 8) in order to derive the statistic distribution. This seems to be paradoxical since the Bai and Ng test is precisely intended to take into account these individual dependencies. However, it is worth noting that Bai and Ng only assume the independence between individual components $e_{i,t}$ of the variable $y_{i,t}$ defined by the exclusion of common components. Thus, in this case, we are far from the cross-sectional independence hypothesis of Im *et al.*

(2003) or Maddala and Wu (1999) on the whole series $y_{i,t}$. From an economic viewpoint, it is thus a hypothesis which can be justified since there is no specification error, especially on the number of common components r. Under the hypothesis of $e_{i,t}$ asymptotic independency, the test statistics based on the estimate components $\hat{e}_{i,t}$ are also asymptotically independent. Thus, the p-values $p^c(i)$ are independently distributed as uniform laws on $[0,1]$. In conclusion, under the hypothesis that all individual components $e_{i,t}$ for $i = 1,...,N$ are $I(1)$, the test statistic Z^c follows a $N(0,1)$ distribution, whatever the panel size N. Note that one can adopt the Choi (2001)'s standardization for panels with large sizes.

In order to test the nonstationarity of the common factors, Bai and Ng (2004) distinguish two cases[7]. When there is only one common factor among the N variables ($r = 1$), they use a standard ADF test in a model with an intercept:

$$\Delta \hat{F}_{1,t} = c + \gamma_{i,0}\hat{F}_{1,t-1} + \gamma_{i,1}\Delta \hat{F}_{1,t-1} + .. + \gamma_{i,p}\Delta \hat{F}_{1,t-p} + v_{i,t} \qquad (23)$$

The corresponding ADF t-statistic, denoted $ADF_{\hat{F}}^c$, has the same limiting distribution as the Dickey-Fuller test for the constant only case. If there are more than one common factors ($r > 1$), Bai and Ng test the number of common independent stochastic trends in these common factors, denoted r_1. Naturally, if $r_1 = 0$, it implies that there are N cointegrating vectors for N common factors, and that all factors are $I(0)$. Individually testing each of the factors for the presence of a unit root generally overstates the number of common trends. So, Bai and Ng propose two statistics based on the r demeaned estimated factors $\hat{F}_{m,t}$ for $m = 1,...,m$. These statistics are similar to those proposed by Stock and Watson (1988). The aim is to test if the real part of the smallest eigenvalue of an autoregressive coefficient matrix is unity. Both statistics involve successive testing of a sequence of hypotheses, much like the tests for the number of cointegrating vectors developed by Johansen (1988). The null hypothesis is defined as $H_0 : r_1 = m$. If the null is rejected, we set $m = m - 1$ and test another time. Otherwise, the estimated number of common trends, denoted $\hat{r}_1$, is equal to m. At the first step, we test the equality between the number of common trends and the number of common factors, *i.e.* $r_1 = r$. The first test statistic, denoted MQ_f, assumes that the nonstationary components are finite order vector-autoregressive processes. The second statistic, denoted MQ_c, allows the unit root process to have more general dynamics.

A similar approach can be adopted in the case of a model including specific deterministic trends ($\alpha_i \neq 0$ in the model 16). The only difference stays in step 1, where $\Delta y_{i,t}$ has to be replaced by $\Delta y_{i,t} - \overline{\Delta y_i}$, with $\overline{\Delta y_i} = (T-1)^{-1}\sum_{t=2}^{T}\Delta y_{i,t}$. In the same way, in the model (19), the variation of the common component must be centered, that is $f_t = \Delta F_t$ must be replaced by $\Delta F_t - \overline{\Delta F_t}$; the same transformation holds for the idiosyncratic component $\Delta e_{i,t} - \overline{\Delta e_{i,t}}$. In this case, under the null hypothesis, the test statistic associated with the common component of step 2, denoted as $ADF_F^\tau(m)$, follows the Dickey-Fuller distribution corresponding to a model including both a constant and a trend. The test statistic associated with the individual component of i, denoted as $ADF_e^\tau(i)$, follows an asymptotic distribution linked to a Brownian bridge and similar to that of the Schmidt and Lee (1991) test.

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[7]In the first working paper (Bai and Ng, 2001), the procedure was the same whatever the number of common factors and was only based on ADF tests.

As argued by Banerjee and Zanghieri (2003), the implementation of the Bai and Ng (2004) test, as it has been described here, clearly illustrates the importance of co-movements across individuals. Indeed, for panels characterized by a strong cross-sectional dependency, the Bai and Ng tests — by taking account the common factors across series — accept the null hypothesis of a unit root in the factors, leading to the conclusion that the series is nonstationary. To conclude, note that the simulations made by Bai and Ng (2004) show that their test gives satisfactory results in terms of size and power, even for moderate panel sample sizes ($N = 20$).

3.2   The Phillips and Sul (2003a) and Moon and Perron (2004a) tests

In contrast to Bai and Ng (2001), Phillips and Sul (2003a) and Moon and Perron (2004a) directly test the presence of a unit root in the observable series $y_{i,t}$. Indeed, they do not proceed to separate tests on the individual and common components. Beyond this fundamental difference, there exists some similarities between the two approaches due to the use of a factor model. We present here in detail the Moon and Perron (2004a) test which allows for the most general specification of the common components.

Moon and Perron (2004a) consider a standard autoregressive process with fixed effects in which residuals follow a factor model. The model structure is thus different from that of Bai and Ng (2001). Considering the same notations as before, their model can be written:

$$y_{i,t} = \alpha_i + y_{i,t}^0 \tag{24}$$

$$y_{i,t}^0 = \phi_i y_{i,t-1}^0 + \mu_{i,t} \tag{25}$$

$$\mu_{i,t} = \lambda_i' F_t + e_{i,t} \tag{26}$$

In a first time, one assumes that the dimension r of the vector F_t is known *a priori* and that the idiosyncratic shocks $e_{i,t} = \sum_{=0}^{\infty} d_{i,j} v_{i,t-j}$, with $v_{i,t}$ *i.i.d.* $(0,1)$ are uncorrelated in the individual dimension. Then, the cross-sectional correlation of $y_{i,t}$ variables is determined by the vector λ_i since $E\left(\mu_{i,t}\mu_{j,t}\right) = \lambda_i' E\left(F_t F_t'\right) \lambda_i$. One tests the null hypothesis of a unit root for all individuals $H_0 : \phi_i = 1$, $\forall i = 1, ..., N$ against $H_1 : \phi_i < 1$ for at least one individual i.

The underlying idea of the Moon and Perron (2004a) approach is the following: it consists on the transformation of the model in order to eliminate the common components of the $y_{i,t}$ series, and on the application of the unit root test on de-factored series. Applying this procedure removes the cross-sectional dependencies and it is then possible to derive normal asymptotic distributions. Thus, normal distributions are obtained, as for Im *et al.* (2003) or Levin and Lin (1992, 1993), but the fundamental difference here is that the test statistics are computed from de-factored data. Thus, they are independent in the individual dimension.

Suppose that we initially have $T + 1$ observations of the $y_{i,t}$ variable. Let Z be the matrix of individual observations of $y_{i,t}$ and Z_{-1} the lagged observations matrix. Finally, let Λ be the (N, r) matrix of factor loadings λ_i and F the common components matrix:

$$\underset{(T,N)}{Z} = \begin{pmatrix} y_{1,2} & .. & y_{N,2} \\ ... & & \\ y_{1,T+1} & & y_{N,T+1} \end{pmatrix} \quad \underset{(T,N)}{Z_{-1}} = \begin{pmatrix} y_{1,1} & .. & y_{N,1} \\ ... & & \\ y_{1,T} & & y_{N,T} \end{pmatrix} \quad \underset{(T,r)}{F} = \begin{pmatrix} F_1' \\ ... \\ F_T' \end{pmatrix}$$

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As previously mentioned, the idea of Moon and Perron (2004a) consists in testing the unit root on the de-factored series. Suppose that the parameters of Λ are known. The de-factored data are written as the projection of Z to the space orthogonal to the factor loadings (*i.e.* the space generated by the columns of Λ). Indeed, to simplify, consider the model (25) under the null hypothesis of a unit root ($\rho_i = 1$) and in the absence of fixed effects ($\alpha_i = 0$), written in a vectorial form:

$$Z = Z_{-1} + F\Lambda' + e$$

where e is the (T, N) idiosyncratic components matrix. Let $Q_\Lambda = I_N - \Lambda(\Lambda'\Lambda)^{-1}\Lambda'$ be the projection matrix to the space orthogonal to the factor loadings. By right-multiplying by Q_Λ, the model can be expressed from de-factored data ZQ_Λ:

$$ZQ_\Lambda = Z_{-1}Q_\Lambda + eQ_\Lambda \tag{27}$$

The projection ZQ_Λ corresponds to de-factored data, while the residuals eQ_Λ do not exhibit, by construction, any cross-sectional correlation. From these de-factored data, the unit root test will be implemented. Moon and Perron construct a test statistic from the pooled estimator[8] ($\phi_i = \phi_j$) of the autoregressive root. More specifically, they consider a corrected pooled estimator, due to the possible cross-units autocorrelation of the residuals eQ_Λ:

$$\widehat{\phi}^+_{pool} = \frac{\text{trace}\left(Z_{-1}Q_\Lambda Z'\right) - NT\lambda_e}{\text{trace}\left(Z_{-1}Q_\Lambda Z'_{-1}\right)} \tag{28}$$

where $\lambda_e = N^{-1}\sum_{i=1}^{N}\lambda_e^i$, λ_e^i is the sum of positive autocovariances of the idiosyncratic components:

$$\lambda_e^i = \sum_{l=1}^{\infty}\sum_{j=0}^{\infty} d_{i,j}d_{i,j+l}$$

From this estimator $\widehat{\phi}^+_{pool}$, Moon and Perron propose two test statistics of the null hypothesis of a unit root, denoted as t_a and t_b respectively. These two statistics converge, assuming T and N tend to infinity and N/T tends to 0.

$$t_a = \frac{T\sqrt{N}\left(\widehat{\phi}^+_{pool} - 1\right)}{\sqrt{2\gamma_e^4/w_e^4}} \xrightarrow[T,N\to\infty]{d} N(0,1) \tag{29}$$

$$t_b = T\sqrt{N}\left(\widehat{\phi}^+_{pool} - 1\right)\sqrt{\frac{1}{NT^2}\text{trace}\left(Z_{-1}QZ'_{-1}\right)\frac{w_e^2}{\gamma_e^4}} \xrightarrow[T,N\to\infty]{d} N(0,1) \tag{30}$$

The quantities w_e^2 and γ_e^4 respectively correspond to the means on N of the individual long-term variances $w_{e,i}^2$ and of squared individual long-term variances $\gamma_{e,i}^4$ of the idiosyncratic component $e_{i,t}$ with $w_{e,i}^2 = \left(\sum_{j=0}^{\infty} d_{i,j}\right)^2$. If the calculated statistic t_a (or t_b) is lower than the normal critical level, the null hypothesis of a unit root is rejected for all individuals.

Note that we have the same $T\sqrt{N}$ convergence rate of the pooled estimator (corrected or no) of the autoregressive root as the one obtained in the Levin and Lin model. This is not surprising since, after having removed the cross unit dependencies in the Phillips and Moon

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[8] This choice is justified by the fact that this estimate simplifies the study of asymptotic distributions, and allows for the analysis of the test under the near unit root local alternative.

model, one obtains on transformed data a model with common autoregressive root (pooled estimator) similar to that of Levin and Lin under the cross-unit independency hypothesis.

Since the idiosyncratic component $e_{i,t}$ is not observed, the quantities w_e^2, λ_e and γ_e^4 are unknown. The definitions (29) and (30) are not directly feasible. In order to implement these tests, two (linked) complements must be added:

1. An estimate of common and idiosyncratic components: more specifically, an estimate $\widehat{Q}_\Lambda$ of the projection matrix to the space orthogonal to the factor loadings.

2. Estimates of the long-term variances ($\widehat{w}_e^2$ and $\widehat{\gamma}_e^4$) and of the positive autocovariances sum ($\widehat{\lambda}_e$), construct from the individual components estimations $\widehat{e}_{i,t}$.

Estimation of the projection matrix

As Bai and Ng, Moon and Perron suggest to estimate Λ by a principal component analysis on the residuals. One begins with the estimation of the residuals $\mu_{i,t}$ from the model (25). To do this, one uses a pooled estimator $\widehat{\phi}_{pool}$ on original individual centered data to account for fixed effects:

$$\widehat{\mu} = \widetilde{Z} - \widehat{\phi}_{pool}\widetilde{Z}_{-1} \tag{31}$$

where $\widetilde{Z} = Q_x Z$ and $Q_x = I_T - T^{-1}l_T l_T'$ where $l_T = (1,...,1)'$, the pooled estimator being defined as $\widehat{\phi}_{pool} = \text{trace}\left(\widetilde{Z}_{-1}'\widetilde{Z}\right)/\text{trace}\left(\widetilde{Z}_{-1}'\widetilde{Z}_{-1}\right)$. From the residuals $\widehat{\mu}$, a principal component analysis is applied. An estimate $\widetilde{\Lambda}$ of the (N, r) matrix of common components coefficients is $\sqrt{N}$ times a matrix whose columns are defined by the r eigenvectors associated with the r largest eigenvalues of the matrix $\widehat{\mu}'\widehat{\mu}$. Moon and Perron propose a re-scaled estimate $\widehat{\Lambda}$:

$$\widehat{\Lambda} = \widetilde{\Lambda}\left(\frac{1}{N}\widetilde{\Lambda}'\widetilde{\Lambda}\right)^{\frac{1}{2}} \tag{32}$$

From this estimate $\widehat{\Lambda}$, one defines an estimate of the projection matrix $\widehat{Q}_\Lambda$ which will allow us to obtain an estimation of the idiosyncratic components:

$$\widehat{Q}_\Lambda = I_N - \widehat{\Lambda}\left(\widehat{\Lambda}'\widehat{\Lambda}\right)^{-1}\widehat{\Lambda}' \tag{33}$$

An estimation of the de-factored data is thus defined by $\widetilde{Z}\widehat{Q}_\Lambda$. One can deduce a pooled corrected estimate $\widehat{\phi}_{pool}^+$ of the autoregressive root only on these de-factored components according to (28), replacing ZQ_Λ by $\widetilde{Z}\widehat{Q}_\Lambda$.

Estimation of the long-term variances

An estimate of the idiosyncratic component is then defined as $\widehat{e} = \widehat{\mu}\widehat{Q}_\Lambda$. Let $\widehat{e}_{i,t}$ be the individual component of the unit i residuals at time t. For each individual $i = 1,...,N$, one defines the residual empirical autocovariance:

$$\widehat{\Gamma}_i(j) = \frac{1}{T}\sum_{t=1}^{T-j}\widehat{e}_{i,t}\widehat{e}_{i,t+j}$$

From $\widehat{\Gamma}_i(j)$, one constructs a kernel-type estimator of the long-term variance and of the positive autocovariances sum as:

$$\widehat{w}_{e,i}^2 = \sum_{j=-T+1}^{T-1} w(q_i, j)\,\widehat{\Gamma}_i(j) \tag{34}$$

$$\widehat{\lambda}_{e,i} = \sum_{j=1}^{T-1} w(q_i, j)\,\widehat{\Gamma}_i(j) \tag{35}$$

where $w(q_i, j)$ is a kernel function and q_i a truncation parameter. Finally, one has to define the estimates of the means of the individual long-term variances:

$$\widehat{w}_e^2 = \frac{1}{N}\sum_{i=1}^{N}\widehat{w}_{e,i}^2 \quad \widehat{\lambda}_e = \frac{1}{N}\sum_{i=1}^{N}\widehat{\lambda}_{e,i} \quad \widehat{\gamma}_e^4 = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{w}_{e,i}^2\right)^2 \tag{36}$$

Let $\widetilde{t}_a$ and $\widetilde{t}_b$ be the statistics defined by equations (29) and (30) substituting w_e^2, λ_e and γ_e^4 by their estimates. $\widetilde{t}_a$ and $\widetilde{t}_b$ converge to normal laws if the kernel functions and truncation parameter q satisfy three hypotheses. Moon and Perron use a QS kernel-type function:

$$w(q_i, j) = \frac{25}{12\pi^2 x^2}\left[\frac{\sin(6\pi x/5)}{6\pi x/5} - \cos\left(\frac{6\pi x}{5}\right)\right] \quad \text{for } x = \frac{j}{q_i}$$

where the optimal truncation parameter q_i is defined as:

$$q_i = 1.3221\left[\frac{4\,\widehat{\rho}_{i,1}^2 T_i}{\left(1 - \widehat{\rho}_{i,1}^2\right)^4}\right]^{1/5}$$

where $\widehat{\rho}_{i,1}^2$ is the first-order autocorrelation estimate of the individual component $\widehat{e}_{i,t}$ of unit i.

Phillips and Sul (2003a) consider a rather more restrictive model than Moon and Perron (2004a) since it contains only one factor independently distributed across time. The common factors vector F_t reduces to a variable θ_t $N.i.d.(0,1)$. Beyond this point, the main difference stays in the approach used to remove the common factors: Phillips and Sul adopt a moment method instead of a principal component analysis. The underlying idea is to test the unit root on orthogonalized data: since these data are independent in the individual dimension, it is possible to apply the first generation unit root tests. Thus, from the estimates of the individual autoregressive roots on orthogonalized data, Phillips and Sul construct various unit root statistics: mean statistics similar to those of Im *et al.* (2003), or statistics based on p-values combinations associated with individual unit root tests.

3.3   The Choi tests

Like Moon and Perron (2004a), Choi (2002) tests the unit root hypothesis using the modified observed series $y_{i,t}$ that allows the elimination of the cross-sectional correlations and the potential deterministic trend components. However, if the principle of the Choi (2002) approach is globally similar to that of Moon and Perron, it differs in two main points. First, Choi considers an error-component model:

$$y_{i,t} = \alpha_i + \theta_t + v_{i,t} \tag{37}$$

$$v_{i,t} = \sum_{j=1}^{p_i} d_{i,j} v_{i,t-j} + \varepsilon_{i,t} \tag{38}$$

where $\varepsilon_{i,t}$ is $i.i.d.\left(0, \sigma_{\varepsilon,i}^2\right)$ and independently distributed across individuals. The time effect θ_t is represented by a weakly stationary process. In this model, in contrast to Bai and Ng (2001) and Moon and Perron (2004a), there exists only one common factor $(r = 1)$ represented by the time effect θ_t. More fundamentally, the Choi model assumes that the individual variables $y_{i,t}$ are equally affected by the time effect (*i.e.* the unique common factor). Here, we have a great difference with Phillips and Sul (2003a): they also consider only one common factor, but they assume a heterogeneous specification of the sensitivity to this factor, like $\lambda_i \theta_t$. Choi justifies his choice by the fact that the logarithmic transformation of the model (37) allows us to introduce such a sensitivity. Moreover in the Choi model, it is possible to test the weak stationarity hypothesis of the θ_t process, while it is not feasible with a heterogenous sensitivity. This is an important point since, from a macroeconomic viewpoint, these time effects are supposed to capture the breaks in the international conjuncture, and nothing guarantees that these effects are stationary.

In the model (37), one tests the null hypothesis of a unit root in the idiosyncratic component $v_{i,t}$ for all individuals, that can be written $H_0 : \sum_{j=1}^{p_i} d_{i,j} = 1$, $\forall i = 1, ..., N$, against the alternative hypothesis that there exists some individuals i such that $\sum_{j=1}^{p_i} d_{i,j} < 1$.

The second difference with the Moon and Perron (2004a) approach, linked to a certain extent to the model specification, stays in the orthogonalization of the individual series $y_{i,t}$ that will be used for a unit root test only on the individual component. To eliminate the cross-sectional correlations, Choi insulates $v_{i,t}$ by eliminating the intercept (individual effect) α_i, but also, and more importantly, the common error term θ_t (time effect). To suppress these deterministic components, Choi use a two-step procedure: the use of the Elliott, Rothenberg et Stock (1996) approach, hereafter ERS, to remove the intercept, and the suppression of the time effect by cross-sectional demeaning. Indeed, when the $v_{i,t}$ component is stationary, OLS provides a fully efficient estimator of the constant term. However, when $v_{i,t}$ is $I(1)$ or presents a near unit root, the ERS approach — that estimates the constant term on quasi-differenced data using GLS — *in fine* allows to obtain better finite sample properties for the unit root tests. For this reason, the Choi (2002) test constitutes, to our knowledge, the first extension of the ERS approach in a panel context. Let us now describe the two steps of the individual series orthogonalization.

Step 1: If one assumes that the largest root of the $v_{i,t}$ process is $1 + c/T$ (near unit root process), for all $i = 1, ..., N$, one constructs two quasi-differenced series $\widetilde{y}_{i,t}$ and $\widetilde{c}_{i,t}$ such that, for $t \geq 2$:

$$\widetilde{y}_{i,t} = y_{i,t} - \left(1 + \frac{c}{T}\right) y_{i,t-1} \qquad \widetilde{c}_{i,t} = 1 - \left(1 + \frac{c}{T}\right) \tag{39}$$

Choi considers the value given by ERS in the case of a model without time trend, that is $c = -7$. Using GLS, one regresses $\widetilde{y}_{i,t}$ on the deterministic variable $\widetilde{c}_{i,t}$. Let $\widehat{\alpha}_i$ be the GLS estimate obtained for each individual i on quasi-differenced data. For T large enough, one has:

$$y_{i,t} - \widehat{\alpha}_i \simeq \theta_t - \theta_1 + v_{i,t} - v_{i,1}$$

whatever the process $v_{i,t}$ being $I(1)$ or near-integrated. Then, one has to eliminate the common component θ_t that can induce correlation across individuals. This is the aim of the second step.

Step 2: Choi suggests to demean $y_{i,t} - \widehat{\alpha}_i$ cross-sectionally and to define a new variable $z_{i,t}$ such that:

$$z_{i,t} = (y_{i,t} - \widehat{\alpha}_i) - \frac{1}{N} \sum_{i=1}^{N} (y_{i,t} - \widehat{\alpha}_i) \tag{40}$$

Indeed, given the previous results, for each individual $i = 1, ..., N$, one can show that:

$$z_{i,t} \simeq (v_{i,t} - \overline{v}_t) - (v_{i,1} - \overline{v}_1)$$

where $\overline{v}_t = (1/N) \sum_{i=1}^{N} v_{i,t}$. Thus, the deterministic components α_i and θ_t are removed from the definition of $z_{i,t}$. The processes $z_{i,t}$ can be considered as independent in the individual dimension since the means $\overline{v}_1$ and $\overline{v}_t$ converge in probability to 0 when N goes to infinity. Then, one has to run a unit root test on the transformed series $z_{i,t}$.

In the case of a model with individual time trends, the approach is similar, but we have to main differences: to obtain the estimates $\widehat{\alpha}_i$ and $\widehat{\gamma}_i$ (the coefficient associated with the individual trend), one regresses the series $\widetilde{y}_{i,t}$ on $\widetilde{c}_{i,t}$ and $\widetilde{d}_{i,t} = 1 - c/T$ using GLS. Then, one chooses the value $c = -13.5$ given by ERS in the case of a model with a time trend. One constructs $z_{i,t}$ such that:

$$z_{i,t} = (y_{i,t} - \widehat{\alpha}_i - \widehat{\gamma}_i t) - \frac{1}{N} \sum_{i=1}^{N} (y_{i,t} - \widehat{\alpha}_i - \widehat{\gamma}_i t)$$

Given the series $\{z_{i,t}\}_{t=2}^{T}$, one runs individual unit root tests without constant, nor trend, whatever the considered model, since all deterministic components have been removed:

$$\Delta z_{i,t} = \rho_i z_{i,t-1} + \sum_{j=1}^{q_i} \beta_{i,j} \Delta z_{i,t-j} + u_{i,t} \tag{41}$$

For a model including a constant, Choi (2002) shows that the Dickey-Fuller t-statistic $t_{\rho_i}^{ERS}$ follows, under $H_{0,i} : \rho_i = 0$, the Dickey-Fuller limiting distribution when T and N tend to infinity. When there is a time trend, this statistic follows a distribution tabulated by ERS. Again, one has to notice that these tests on time series, while being more powerful than the standard ADF tests, are less powerful than panel tests. For this reason, Choi suggests panel test statistics based on the individual $t_{\rho_i}^{ERS}$ statistics which are independent. Using its previous work (Choi, 2001), Choi (2002) proposes three statistics based on combinations of significance levels of individual tests:

$$P_m = -\frac{1}{\sqrt{N}} \sum_{i=1}^{N} [\ln(p_i) + 1] \tag{42}$$

$$Z = -\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \Phi^{-1}(p_i) \tag{43}$$

$$L^* = \frac{1}{\sqrt{\pi^2 N/3}} \sum_{i=1}^{N} \ln\left(\frac{p_i}{1-p_i}\right) \tag{44}$$

where p_i is the significance level of the statistic $t_{\rho_i}^{ERS}$, and $\Phi(.)$ is the standard cumulative normal distribution function. Under the unit root null hypothesis for all individuals, Choi (2002) shows that these three statistics converge to a standardized normal law when T and N tend to infinity. The decision rules are the following. For the transformed Fisher statistic P_m, if the realization is greater than the standard normal law level (1.64 at the 5% significance level), one rejects H_0. For the two statistics Z and L^*, if the realization is lower than the normal law level (-1.64 at the 5% significance level), H_0 is rejected.

As for the works of Choi (2001) or Maddala and Wu (1999), the main difficulty in this approach stays in the fact that one has to simulate, using bootstrap methods, the p-values p_i used for the construction of the statistics P_m, Z and L^*, specially if one uses the ERS distributions. Choi uses the MacKinnon (1994) methodology which can be described in three steps. One simulates under H_0 data on T periods according to the process $x_t = x_{t-1} + u_t$ where u_t is $N.i.d.(0,1)$. It is recommended to consider realizations of x on a longer period and to retain only the last T observations to avoid sensitivity to initial conditions.

Step 1: For different values of T (Choi considers $T = 30, 50, 75, 100, 250, 500, 1000$), generate $\{x_t\}_{t=1}^{T}$ I times. For each series, one computes the $t_{\rho_i}^{ERS}$ statistic in the ADF model (41). Given the I realizations $\widehat{t}_{\rho_i}^{ERS}$, one constructs 399 equally spaced percentiles.

Step 2: One repeats S times the first step (Choi chooses $S = 50$): for the seven considered values of T, one obtains S values for each of the 399 percentiles. Let $q_j^p(T)$ be the value of the percentile obtained at the jth simulation ($j = 1, ..., 350$) for $p = 0.0025, 0.0050, ..., 0.9975$ for a size T.

Step 3: For each level p and each size T, one estimates, using GLS, the following equation given the S disposable observations:

$$q_j^p(T) = \eta_\infty^p + \eta_1^p T^{-1} + \eta_2^p T^{-2} + \varepsilon_j \quad j = 1, ..., S$$

For a given size T, one obtains 399 realizations of the GLS estimates of the parameters η_∞^p, η_1^p and η_2^p. For example, for $T = 30$, Choi obtains $\widehat{\eta}_\infty^{0.05} = -1.948$, $\widehat{\eta}_1^{0.05} = -19.36$ and $\widehat{\eta}_2^{0.05} = 143.4$ for $T = 100$. Therefore, one can establish the 5% percentile of the $t_{\rho_i}^{ERS}$ statistic such that:

$$-1.948 - \frac{19.36}{100} + \frac{143.4}{100^2} = -2.1273$$

Thus, for each size T, one obtains 399 values for the percentiles for $p = 0.0025, 0.0050, ..., 0.9975$. Using linear interpolation, one derives the p-value p_i of the $t_{\rho_i}^{ERS}$ test statistic that will be used in the construction of the P_m, Z and L^* test statistics.

It is noteworthy that the obtained p-values are somewhat sensitive to the law used to generate the process u_t. For high T sizes, to wrongly suppose the normality of the residuals u_t would not matter. However, for low T sizes, the p-values obtained under the normality hypothesis may not be accurate, if the underlying distribution is quite different from the normal law.

3.4 The Pesaran tests

Pesaran (2003) proposes a different approach to deal with the problem of cross-sectional dependencies. He considers a one-factor model with heterogeneous loading factors for residuals, as in Phillips and Sul (2003a). However, instead of basing the unit root tests on deviations from the estimated common factors, he augments the standard Dickey-Fuller or Augmented Dickey-Fuller regressions with the cross section average of lagged levels and first-differences of the individual series. If residuals are not serially correlated, the regression used for the ith country is defined as:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + c_i \overline{y}_{t-1} + d_i \Delta \overline{y}_t + v_{i,t} \tag{45}$$

where $\overline{y}_{t-1} = (1/N) \sum_{i=1}^{N} y_{i,t-1}$ and $\Delta \overline{y}_t = (1/N) \sum_{i=1}^{N} \Delta y_{i,t}$. Let us denote $t_i(N,T)$ the t-statistic of the OLS estimate of ρ_i. The Pesaran's test is based on these individual cross-sectionally augmented ADF statistics, denoted CADF. A truncated version, denoted CADF*, is also considered to avoid undue influence of extreme outcomes that could arise for small T samples. In both cases, the idea is to build a modified version of IPS t-bar test based on the average of individual CADF or CADF* statistics (respectively denoted CIPS and CIPS*, for cross-sectionally augmented IPS).

$$CIPS = \frac{1}{N} \sum_{i=1}^{N} t_i(N,T) \qquad CIPS^* = \frac{1}{N} \sum_{i=1}^{N} t_i^*(N,T) \tag{46}$$

where the truncated CADF statistic is defined as:

$$t_i^*(N,T) = \begin{cases} K_1 & \text{si } t_i(N,T) \leq K_1 \\ t_i(N,T) & \text{si } K_1 < t_i(N,T) < K_2 \\ K_2 & \text{si } t_i(N,T) \geq K_2 \end{cases} \tag{47}$$

The constants K_1 and K_2 are fixed such that the probability that $t_i(N,T)$ belongs to $[K_1, K_2]$ is near to one. In a model with intercept only, the corresponding simulated values are respectively -6.19 and 2.61 (Pesaran, 2003).

All the individual CADF (or CADF*) statistics have similar asymptotic null distributions which do not depend on the factor loadings. But they are correlated due to the dependence on the common factor. Therefore, it is possible to build an average of individual CADF statistics, but standard central limit theorems do not apply to these CIPS or CIPS* statistics. Pesaran shows that, even if it is not normal, the null asymptotic distribution of the truncated version of the CIPS statistic exists and is free of nuisance parameter. He proposes simulated critical values of CIPS and CIPS* for various samples sizes. Pesaran also uses Fisher type tests based on the significant levels of individual CADF statistics, as those proposed by Maddala and Wu (1999) or Choi (2001). Given the reasons mentioned above, such statistics do not have standard distributions. Finally, this approach readily extends to serially correlated residuals. For an AR(p) error specification, the relevant individual CADF statistics are computed from a pth order cross-section/time series augmented regression:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + c_i \overline{y}_{t-1} + \sum_{j=0}^{p} d_{i,j} \Delta \overline{y}_{t-j} + \sum_{j=0}^{p} \beta_{i,j} \Delta y_{i,t-j} + \mu_{i,t} \tag{48}$$

4 A second generation of tests: other approaches

There is a second approach to model the cross-sectional dependencies, which is more general than those based on dynamic factors models or error component models. It consists in imposing few or none restrictions on the covariance matrix of residuals. It is in particular the solution adopted in O'Connell (1998), Maddala and Wu (1999), Taylor and Sarno (1998), Chang (2002, 2004). Such an approach raises some important technical problems. With cross-sectional dependencies, the usual Wald type unit root tests based on standard estimators have limit distributions that are dependent in a very complicated way upon various nuisance parameters defining correlations across individual units. There does not exist any simple way to eliminate these nuisance parameters.

The first attempt to deal with this problem was done in O'Connell (1998). He considers a covariance matrix similar as that would arise in an error component model with mutually independent random time effects and random individual effects. However, this specification of cross-sectional correlations remains too specific to be widely used. Another attempt was proposed in Maddala and Wu (1999). They propose to way out the problem by using a bootstrap method to get the empirical distributions of the LL, IPS or Fisher's type test statistics in order to make inferences. Their approach is technically difficult to implement since it requires bootstrap methods for panel data. Besides, as pointed out by Maddala and Wu, the bootstrap methods result in a decrease of the size distortions due to the cross-sectional correlations, although it does not eliminate them. So, the bootstrap versions of the first generation tests performs much better, but do not provide the validity of using bootstrap methodology. More recently, a second generation bootstrap unit root test has been proposed by Chang (2004). He considers a general framework in which each panel is driven by a heterogeneous linear process, approximated by a finite order autoregressive process. In order to take into account the dependency among the innovations, Chang proposes a unit root test based on the estimation on the entire system of N equations. The critical values are then computed by a Bootstrap method.

Another solution consists in using the instrumental variable (IV thereafter) to solve the nuisance parameter problem due to cross-sectional dependency. It is the solution adopted in Chang (2002). The Chang (2002) testing procedure is as follows. In a first step, for each cross-section unit, he estimates the autoregressive coefficient from a usual ADF regression using the instruments generated by an integrable transformation of the lagged values of the endogenous variable. He then constructs N individual t-statistics for testing the unit root based on these N nonlinear IV estimators. For each unit, this t-statistic has limiting standard normal distribution under the null hypothesis. In a second step, a cross-sectional average of these individual unit test statistics is considered, as in IPS.

Let us consider the following ADF model:

$$\Delta y_{i,t} = \alpha_i + \rho_i y_{i,t-1} + \sum_{j=1}^{p_i} \beta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t} \tag{49}$$

where $\varepsilon_{i,t}$ are $i.i.d.\left(0, \sigma_{\varepsilon_i}^2\right)$ across time periods, but are allowed to be cross-sectionally de-

pendent. To deal with this dependency, Chang uses the instrument generated by a nonlinear function $F(y_{i,t-1})$ of the lagged values $y_{i,t-1}$. This function $F(.)$ is called the Instrument Generating Function (IGF thereafter). It must be a regularly integrable function which satisfies $\int_{-\infty}^{\infty} xF(x)\,dx \neq 0$. This assumption can be interpreted as the fact that the nonlinear instrument $F(.)$ must be correlated with the regressor $y_{i,t-1}$.

Let $x_{i,t}$ be the vector of lagged demeaned differences $(\Delta y_{i,t-1}, ..., \Delta y_{i,t-p_i})'$ and $X_i = (x_{i,p_i+1}, ..., x_{i,T})'$ the corresponding (T, p_i) matrix. Define $y_{l,i} = (y_{i,p_i}, ..., y_{i,T-1})'$ the vector of lagged values and $\varepsilon_i = (\varepsilon_{i,p_i+1}, ..., \varepsilon_{i,T})'$ the vector of residuals. Under the null, the nonlinear IV estimator of the parameter ρ_i, denoted $\widehat{\rho}_i$, is defined as:

$$
\begin{aligned}
\widehat{\rho}_i &= \left[F(y_{l,i})'\, y_{l,i} - F(y_{l,i})'\, X_i\, (X_i'X_i)^{-1}\, X_i'y_{l,i} \right]^{-1} \\
&\quad \left[F(y_{l,i})'\, \varepsilon_i - F(y_{l,i})'\, X_i\, (X_i'X_i)^{-1}\, X_i'\varepsilon_i \right]
\end{aligned}
\tag{50}
$$

The variance of this IV estimator is:

$$
\begin{aligned}
\widehat{\sigma}_{\widehat{\rho}_i}^2 &= \widehat{\sigma}_{\varepsilon_i}^2 \left[F(y_{l,i})'\, y_{l,i} - F(y_{l,i})'\, X_i\, (X_i'X_i)^{-1}\, X_i'y_{l,i} \right]^{-2} \\
&\quad \left[F(y_{l,i})'\, F(y_{l,i}) - F(y_{l,i})'\, X_i\, (X_i'X_i)^{-1}\, X_i'F(y_{l,i}) \right]
\end{aligned}
\tag{51}
$$

where $\widehat{\sigma}_{\varepsilon_i}^2 = (1/T)\sum_{t=1}^{T}\widehat{\varepsilon}_{i,t}^2$ denotes the usual variance estimator. Chang shows that the t-ratio used to test the unit root hypothesis, denoted Z_i, asymptotically converges to a standard normal distribution if a regularly integrable function is used as an IGF.

$$
Z_i = \frac{\widehat{\rho}_i}{\widehat{\sigma}_{\widehat{\rho}_i}} \xrightarrow[T\to\infty]{d} N(0,1) \quad \text{for } i = 1, ..., N
\tag{52}
$$

This asymptotic Gaussian result is very unusual and entirely due to the nonlinearity of the IV. Chang provides several examples of regularly integrable IGFs. Moreover, the asymptotic distributions of individual Z_i statistics are independent across cross-sectional units. So, panel unit root tests based on the cross-sectional average of these individual independent statistics can be implemented. Chang proposes an average IV t-ratio statistic, denoted S_N and defined as:

$$
S_N = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} Z_i
\tag{53}
$$

The factor $N^{-1/2}$ is used just as a normalization factor. In an asymptotically balanced panel in a weak sense and more over in balanced panel as in our case, this statistic has a limit standard normal distribution. This result appears similar to those obtained under the cross-sectional independence assumption, but the limit theories are very different in both cases. Finally, the model with individual effects can be analyzed similarly using demeaned data.

This approach is very general and has good finite sample properties (see Chang, 2002). According to the Chang's simulations, the IV nonlinear test performs better than the IPS tests in terms of both finite sample sizes and powers. However, a recent paper of Im and Pesaran (2003) shows that the asymptotic independence of the IV non linear t-statistics is valid only

under the condition that $N \ln(T)/\sqrt{T} \to 0$ when N and T tend to infinity. The authors remark that this condition is unlikely to hold in practice except when N is small. Besides, their simulations based on common factor model, show that this test suffers from very large size distortions even in the case previously mentioned, i.e. when N is very small relative to T.

5   Conclusion

This survey gives an overview of the main unit root tests in the econometrics of panel data. It puts forward that two directions of research have been developed since the seminal work of Levin and Lin (1992), leading to two generations of panel unit root tests. The first one concerns heterogeneous modellings with the contributions of Im, Pesaran and Shin (1997), Maddala and Wu (1999), Choi (2001) and Hadri (2000). The second area of research, more recent, aims to take into account the cross-sectional dependencies. This last category of tests is still under development, given the diversity of the potential cross-sectional correlations.

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